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Event-triggered robust adaptive control for uncertain nonlinear systems preceded by actuator dead-zone

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Abstract It is both theoretically and practically important to investigate the problem of event-triggered adaptive tracking control for a class of uncertain nonlinear systems subject to actuator dead-zone, which aims at reducing communication rate and compensating actuator nonlinearity simultaneously. In this paper, to handle such a problem, an event-trigger based adaptive compensation scheme is proposed for the system preceded by actuator dead-zone. The challenges of this work can be roughly classified into two categories: how to compensate the nonsmooth dead-zone nonlinearity and how to eliminate the quantization signal effects caused by event-triggered strategy. To resolve the first challenge, a new decomposition of dead-zone mathematical model is employed so that dead-zone nonlinearity can be successively compensated by using robust approach. In addition, an adaptive controller and its triggering event are co-designed based on the relative threshold strategy, such that an asymptotic tracking performance can be ensured. The proposed scheme is proved to guarantee the globally bounded of all closed-loop signals and the asymptotic convergence performance of tracking error toward zero. The simulation results illustrate the effectiveness of our proposed control scheme.

Keywords Event-triggered · Adaptive · Backstepping · Dead-zone

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1 Introduction

In practical Networked Control Systems (NCSs), transmissions between sensors, controllers, and actuators are provided by a shared communication network. Due to the limitation of communication channel bandwidth and computation abilities, the problems of energy, communication, and computation constraints ubiquitously exist during signal transmission. Besides inherent such problems in hybrid system, nonlinearities such as dead-zone may also occur in practical mechanisms, causing severe performance deterioration and sometimes leading to instability. Thus, it is of great significance to investigate the problem of reducing communication rate and compensating actuator dead-zone for uncertain nonlinear systems.

In recent researches, the event-triggered control scheme has received considerable attention for its ability in deciding which control task is executed at any given instant, such that the communication rate in the NCSs can be reduced, which has been developed in [6,7,9,10,20,21,24,32,36]. In [24], an event-triggered scheduling algorithm is employed to execute the control task whenever some corresponding errors become magnitude, while the stabilizing properties of such control strategy are set, which are further applied to the nonlinear systems by designing a novel framework for event-triggered stabilization in [10,21]. However, it is worthy to point out that the mentioned results are based on the input-to-state stability (ISS) assumption, which is difficult to satisfy since one cannot find a suitable controller to guarantee an ISS assumption with certain measurement errors even for some affine nonlinear system, as described in [8]. To remove the ISS assumption, in [1,2], a controller and its triggering event are co-designed for the complex networks of nonlinear dynamic systems. The results are finally extended to combine with adaptation mechanism by proposing an event-triggered adaptive control scheme for uncertain nonlinear systems, as shown in [31]. Such proposed scheme successfully removes the ISS assumption and is available in handling the system uncertainties. What's more, numerous results concerning about event-triggered control have been investigated, such as the output feedback control problem [35], the Markov jump systems control problem [5,22,23], and the fault detection problem [15]. However, most of the existing studies about event-triggered tracking control are commonly based on another assumption that actuator effects of dynamic systems are neglected, while in practice, nonlinearities such as dead-zone occurred in actuators are of great importance and necessity to be compensated because it would deteriorate the performance of the system regularly.

For the isolated (without considering network control) systems preceded by actuator dead-zone, adaptive compensation control schemes are developed in [11,12,25,29,30]. In [16,17,33,34], based on a direct decomposition method (DDM), dead-zone model is regarded as an external bounded disturbance effect that can be eliminated by using robust approach. Apart from the DDM, another effective control scheme named adaptive inverse compensation method (AICM) is proposed to estimate unknown dead-zone parameters online, as shown in [25]. Different from the DDM, the AICM can absolutely cancel dead-zone effects by establishing an inverse compensator through parameters identification. Furthermore, the AICM approach is utilized to construct an advance adaptive controller for uncertain nonlinear systems preceded by dead-zone nonlinearity in [37]. But the mentioned results generally suppose the input of the actuator is smooth enough, which cannot be guaranteed under hybrid system since the input signals are quantized. Such constraint is removed in [18] by proposing an adaptive neural network (NN) tracking control scheme for a discrete-time system with dead-zone input. However, to the best knowledge of authors, no result has been reported to investigate the problem of event-triggered adaptive tracking control for the NCSs preceded by dead-zone input.

Through tremendous studies, the most challenge for solving such a problem is that we should take both quantized transmission signal effects and dead-zone behavior into account when develop an adaptive controller. In addition, it is of great difficulty to find an appropriate design method to accommodate such systems and finally ensure an asymptotic tracking performance. To overcome these troubles, in this paper, a new adaptive controller and its event-triggered mechanism are proposed for the nonlinear systems preceded by dead-zone nonlinearity, which are based on robust approach and adaptive control scheme. The contributions of the proposed scheme are summarized as below:

1. The present available results concerning about the event-triggered strategy, for example, [9,21,24,31], are trying to solve the problem of energy consumption and communication constraints, but the existence of actuator dead-zone is always ignored. In this paper, an event-triggered controller is proposed for the hybrid systems preceded by the dead-zone actuator, which is feasible to reduce communication rate and eliminate the effect of dead-zone behavior simultaneously.
2. Since the input of the actuator has been quantized by utilizing event-triggered strategy, the inverse compensation scheme in [37] is not applicable to the hybrid system with discrete dead-zone input. To remove this constraint, a new decomposition model of dead-zone is employed and then a dynamic control coefficient is brought, which can be addressed by robust approach.
3. Note that there is no result recently available in developing an adaptive control scheme to ensure both the asymptotic tracking performance and the

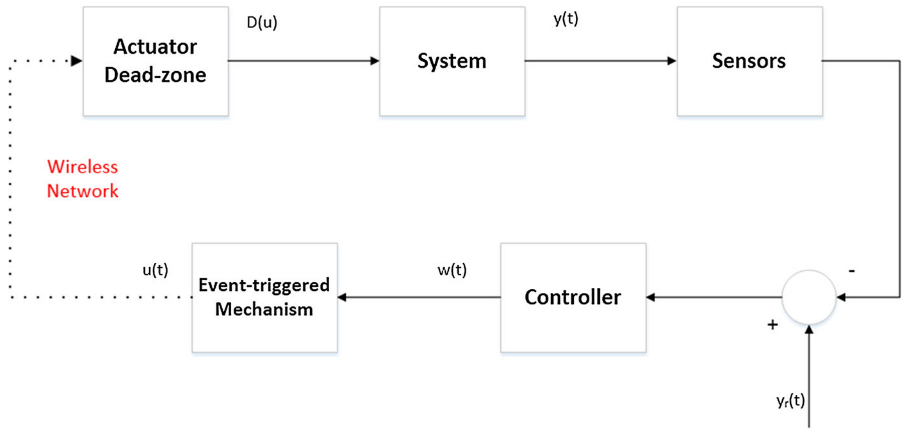


Fig. 1 Event-triggered tracking control scheme for a net-based system

transient tracking performance for event-trigger based systems. Such restriction is successfully removed by our proposed strategy, whose asymptotic tracking performance can be rigorously proved by Barbalat's lemma and transient performance can be improved by tuning design parameters.

The outline of this paper is organized as below. In Sect. 2, a statement about the control problem is presented. In Sect. 2, we propose a new dead-zone decomposition and the objective controller is designed, and the stability analysis is given at the end of this section. The simulations for a specific nonlinear system in the presence of the mentioned problem are shown in Sect. 3, which verify our assumptions. Finally, conclusion and further study for this paper are given in Sect. 4.

1.1 Nonlinear system model

Considering a class of uncertain nonlinear systems preceded by actuator dead-zone as follows.

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \varphi_i^T(x_1, x_2, \ldots, x_i)\theta, i = 1, 2, \ldots, n-1 \\ \dot{x}_n &= \varphi_0(x) + \varphi_n^T(x)\theta + D(u) \\ y &= x_1, \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n] \in \Re^n$ are state variables, $y \in \Re$ is the system output, and $u \in \Re$ is the control input. φ_i for $i = 0, 1, \ldots, n$ are known smooth nonlinear functions. vector $\theta \in \Re^n$ contains unknown structural parameters. $D(u)$ denotes the actuator dead-zone nonlinearity.

Remark 1 Comparing with the existing literature [31] on which the event-triggered adaptive control design is focused, this paper considers the problem of event-triggered adaptive tracking control for uncertain nonlinear systems with actuator dead-zone nonlinearity, as shown in Fig. 1. The techniques of new dead-zone decomposition model and event-triggered strategy are employed in the controller design to, respectively, eliminate actuator dead-zone effect and reduce communication rate.

To handle the mentioned problem and finally achieve a desired tracking performance, the control objectives of this paper are summarized as follows:

– All the closed-loop signals are globally bounded.
– The tracking error $e(t)$ asymptotically converges to zero, i.e., $\lim_{t\to\infty}(y - y_r) = 0$.

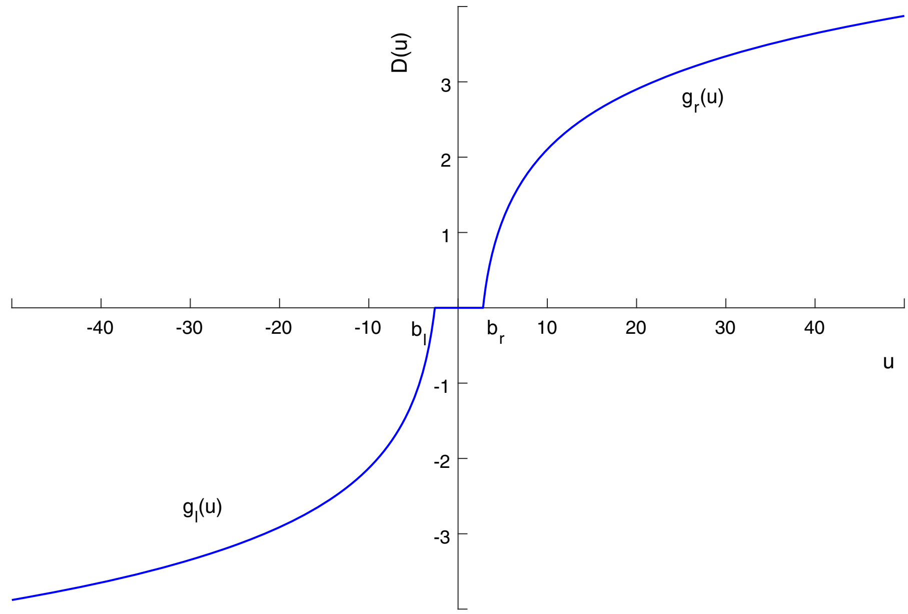
The following assumption is needed for our control design, as pointed out in [13,37].

Assumption 1 The reference signal y_r and its first n-th order derivatives $y_r^{(i)}(i = 1, 2, \ldots, n)$ are known, bounded, and piecewise continuous.

1.2 Dead-zone model

As previous descriptions, the actuator of physical system may suffer from dead-zone nonlinearity, as shown in Fig. 1. For the visualization, the generalized dead-zone model is shown in Fig. 2. Based on the mean value theorem, the following equations are obtained as

Fig. 2 Dead-zone model



$$g_r(u) - 0 = \frac{\partial g_r(u)}{\partial u}\Big|_{u=u_{r0}}(u - b_r)$$
$$g_l(u) - 0 = \frac{\partial g_l(u)}{\partial u}\Big|_{u=u_{l0}}(u - b_l) \quad (2)$$

Thus, the generalized dead-zone dynamic model is computed as

$$D(u) = \begin{cases} \rho_r(t_r)(u - b_r) & u \geq b_r \\ 0 & b_l < u < b_r \\ \rho_l(t_l)(u - b_l) & u \leq b_l \end{cases}, \quad (3)$$

where $b_l \leq 0, b_r \geq 0$ are the unknown constants, normally, the break points $|b_r| \neq |b_l|$. Besides, the unknown constants $\rho_r(t_r)$ and $\rho_l(t_l)$ are depended on the time values t_r and t_l, which satisfy

$$\rho_r(t_r) = \frac{\partial g_r(u)}{\partial u}\Big|_{u(t)=u(t_r)}$$
$$\rho_l(t_l) = \frac{\partial g_l(u)}{\partial u}\Big|_{u(t)=u(t_l)}, \quad (4)$$

where $u(t_r) \geq b_r, u(t_l) \leq b_l$.

Assumption 2 The functions $g_r(u), g_l(u)$ are smooth, and there exist unknown positive constants b_r, b_l, such that $0 < \rho_r(t_r) < +\infty,\ \forall u \in [b_r, +\infty)$ and $0 < \rho_l(t_l) < +\infty,\ \forall u \in (-\infty, b_l]$.

2 Event-triggered robust adaptive control design

2.1 Relative threshold event-triggered strategy

As described in [31], the fixed threshold strategy originally presented in [9] is more effective in the NCSs while the value of control signal u is huge enough. However, it is difficult to choose an appropriate threshold m to obtain a desirable control result when the control signal u approaches zero, such constraint is removed by proposing a relative threshold event-triggered strategy in [24]. In this paper, the control signal is assumed small enough; thus, it is more effective to choose the relative threshold strategy to design an event-triggered adaptive controller, such that better system performance can be achieved. The measurement error $e(t)$ and the triggering event are given as:

$$e(t) = \varpi(t) - u(t), \forall t \in [t_k, t_{k+1}) \quad (5)$$

$$u(t_{k+1}) = \begin{cases} \varpi(t_k) & |e(t)| \geq \delta|u(t)| + m \\ u(t_k) & others \end{cases}, \quad (6)$$

where $\varpi(t)$ and $u(t)$ are the adaptive controller and the real control signal, respectively. $t_k (k \in Z^+)$ denotes the controller update time, and $|e(t)| \geq \delta|u(t)| + m$ is treated as the triggering event, where $0 < \delta < 1$ and $m > 0$. From the event-triggered control scheme in (5)–(6), we discover that the control signal $u(t)$ will be updated as $\varpi(t_k)$ at the time of the event-triggering, i.e., $|e(t)| \geq \delta|u(t)| + m$, or it will keep the previous value as $u(t_k)$.

Lemma 1 *During the interval* $[t_k, t_{k+1})$*, the input signal* $u(t)$ *in the NCSs can be expressed as* $u(t) = \frac{\varpi(t)}{1+\lambda_1(t)\delta} - \frac{\lambda_2(t)m}{1+\lambda_1(t)\delta}$.

Proof Define two continuous time-varying parameters $\lambda_1(t), \lambda_2(t)$, which satisfy $|\lambda_1(t)| \leq 1$ and $|\lambda_2(t)| \leq 1$, respectively. From (6), we have $|\varpi(t) - u(t)| \leq \delta|u(t)| + m$, then $\varpi(t) - u(t) = \lambda_1(t)\delta u(t) + \lambda_2(t)m$ is obtained. Hence, during the interval $[t_k, t_{k+1})$, $u(t) = \frac{\varpi(t)}{1+\lambda_1(t)\delta} - \frac{\lambda_2(t)m}{1+\lambda_1(t)\delta}$ can be proved. □

2.2 Decomposition of dead-zone model

To make a compensation for dead-zone effect in the actuator of practical mechanical systems, the following decomposition of dead-zone nonlinearity is employed.

$$D(u) = c(u)u + d(u) \tag{7}$$

By comparing with the Eq. (3), the following coefficients are obtained as

$$c(u) = \begin{cases} \rho_r(t_r) & u \geq b_r \\ 0 & b_l < u < b_r \\ \rho_l(t_l) & u \leq b_l \end{cases} \tag{8}$$

$$d(u) = \begin{cases} -\rho_r(t_r)b_r & u \geq b_r \\ 0 & b_l < u < b_r \\ -\rho_l(t_l)b_l & u \leq b_l \end{cases} \tag{9}$$

Lemma 2 *In the dead-zone decomposition* (7)*, the dynamic control coefficient* $c(u)$ *and the disturbance-like term* $d(u)$ *satisfy that*

$$0 < \underline{c} \leq c(u) \leq \bar{c} < +\infty \tag{10}$$

$$\underline{c} = min\{\rho_r(t_r), \rho_l(t_l)\},\ \bar{c} = max\{\rho_r(t_r), \rho_l(t_l)\} \tag{11}$$

$$|d(u)| \leq max\{\rho_r(t_r)b_r, -\rho_l(t_l)b_l\} = \varepsilon \tag{12}$$

where $\underline{c}, \bar{c}, \varepsilon$ *are unknown positive constants.*

Remark 2 In the event-trigger based control system, the real control signal $\varpi(t)$ will be quantized in the event-triggered mechanism, and the quantized control signal $u(t)$ is transmitted to the actuator over the networks successively, as shown in Fig. 1. Because of such quantized signal, the proposed inverse function scheme in [37] is not applicable to compensate for the actuator dead-zone. To remove such constraint, a new decomposition of dead-zone dynamic model is employed in (7), and the bounds of the relevant coefficients $c(u)$ and $d(u)$ are obtained in (11) and (12), respectively.

Remark 3 It is important to point out that the dynamic coefficient $c(u)$ is included in the dead-zone decomposition, which adds a nontrivial task to design a controller for compensating dead-zone effects, since $c(u)$ contains unpredictability. Besides, the disturbance-like term $d(u)$ may impose damaging effects on the system performance and even results in instability of the system.

2.3 Event-triggered robust adaptive control scheme

Based on tuning function scheme, an event-triggered adaptive control design is proposed. The details of Step 1 to Step n are elaborated, and the error variables are firstly defined as

$$z_1 = x_1 - y_r$$
$$z_i = x_i - \alpha_{i-1} - y_r^{(i-1)}, i = 2, 3, \ldots, n \tag{13}$$

Step 1 The derivative of the first error variable z_1 is obtained as

$$\dot{z}_1 = z_2 + \alpha_1 + \varphi_1^T(x_1)\theta \tag{14}$$

Define a Lyapunov function candidate as $V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$, where Γ is a symmetric positive definite matrix. The virtual controller α_1 and tuning function τ_1 are designed as

$$\alpha_1 = -c_1z_1 - w_1^T\hat{\theta} \tag{15}$$
$$\tau_1 = w_1z_1 \tag{16}$$
$$w_1 = \varphi_1(x_1), \tag{17}$$

where c_1 is a positive design parameter and $\theta = \hat{\theta} + \tilde{\theta}$, $\hat{\theta}$ is the estimation of θ. Then, the derivative of V_1 can be rewritten as

$$\dot{V}_1 = -c_1z_1^2 + z_1z_2 + \tilde{\theta}^T(\tau_1 - \Gamma^{-1}\dot{\hat{\theta}}) \tag{18}$$

The second term z_1z_2 will be eliminated at the next step.

Step 2 As previously analyzed, the virtual controller α_2 and tuning function τ_2 are given as

$$\alpha_2 = -z_1 - c_2 z_2 - w_2^T \hat{\theta} + \frac{\partial \alpha_1}{\partial x_1} x_2 + \frac{\partial \alpha_1}{\partial y_r} \dot{y}_r + \frac{\partial \alpha_1}{\partial \hat{\theta}} \Gamma \tau_2 \tag{19}$$

$$\tau_2 = \tau_1 + w_2 z_2 \tag{20}$$

$$w_2 = \varphi_2 - \frac{\partial \alpha_1}{\partial x_1} \varphi_1 \tag{21}$$

The derivative of the second Lyapunov function candidate V_2 is computed as

$$\dot{V}_2 = -\sum_{i=1}^{2} c_i z_i^2 + z_2 z_3 + \tilde{\theta}^T (\tau_2 - \Gamma^{-1} \dot{\hat{\theta}}) + \frac{\partial \alpha_1}{\partial \hat{\theta}} z_1 (\Gamma \tau_2 - \dot{\hat{\theta}}) \tag{22}$$

Step i, $(i = 3, \ldots, n)$ The corresponding parameters of the ith step are summarized as

$$\alpha_i = -z_{i-1} - c_i z_i - w_i^T \hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \Gamma \tau_i + \sum_{j=1}^{i-1} \left[\frac{\partial \alpha_{i-1}}{\partial x_j} x_{j+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(j-1)}} y_r^{(j)} \right] + \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \Gamma w_i z_j \tag{23}$$

$$\tau_i = \tau_{i-1} + w_i z_i = \left(w_1 \cdots w_i \right) \begin{pmatrix} z_1 \\ \vdots \\ z_i \end{pmatrix} \tag{24}$$

$$w_i = \varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \varphi_j, \tag{25}$$

where c_i are positive design parameters. The update law in the final step is chosen as

$$\dot{\hat{\theta}} = \Gamma \tau_n \tag{26}$$

With the event-triggered mechanism (5)–(6), the adaptive controller $\varpi(t)$ is finally established by our design scheme.

$$\varpi(t) = \hat{\kappa}^T \nu, \tag{27}$$

where $\hat{\kappa} = (\hat{\kappa}_1, \hat{\kappa}_2, \hat{\kappa}_3)^T$ is the estimation of $\kappa = \left(\frac{1+\delta}{\underline{c}}, \frac{(1+\delta)\varepsilon}{\underline{c}}, \frac{(1+\delta)\bar{m}}{\underline{c}} \right)^T$, $\bar{m} > \frac{m}{1-\delta}$. $\hat{\kappa}$ is updated by the following update law

$$\dot{\hat{\kappa}} = -\Gamma_\kappa \nu z_n, \tag{28}$$

where Γ_κ is a symmetric positive definite matrix.

In addition, $\nu = [\nu_1, \nu_2, \nu_3]^T$ is chosen as

$$\nu_1 = -\frac{z_n \rho^2}{\sqrt{z_n^2 \rho^2 + \epsilon_1(t)^2}}, \quad \nu_2 = -\frac{z_n}{\sqrt{z_n^2 + \epsilon_2(t)^2}}, \quad \nu_3 = -\frac{z_n}{\sqrt{z_n^2 + \epsilon_3(t)^2}} \tag{29}$$

$$\rho = \varphi_0 - \alpha_n - y_r^{(n)}, \tag{30}$$

where $\epsilon_i(t)$ for $i = 1, 2, 3$ are chosen as positive integrable functions, such that $\int_0^\infty \epsilon_i(t) \mathrm{d}t < +\infty$.

Lemma 3 *Note that the inequality $\hat{\kappa} \nu z_n \leq 0$ can be always satisfied in the case of $\hat{\kappa}_i(0) > 0$ for $i = 1, 2, 3$.*

Proof By substituting (29) into (28), $\dot{\hat{\kappa}}_i(t) \geq 0$ is obtained.Via the integral operation, $\hat{\kappa}_i(t) \geq 0$ can be deduced when $\hat{\kappa}_i(0) \geq 0$ is chosen. With the definition of ν in (29), we have

$$\hat{\kappa} \nu z_n = -\hat{\kappa}_1 \frac{z_n^2 \rho^2}{\sqrt{z_n^2 \rho^2 + \epsilon_1(t)^2}} - \hat{\kappa}_2 \frac{z_n^2}{\sqrt{z_n^2 + \epsilon_2(t)^2}} - \hat{\kappa}_3 \frac{z_n^2}{\sqrt{z_n^2 + \epsilon_3(t)^2}} \leq 0 \tag{31}$$

Proof has been completed. □

Remark 4 The property $\hat{\kappa} \nu z_n \leq 0$ is so constructed for our newly proposed control framework. The reason is that for any value of $z_n(t)$, the dynamic control coefficient $c(u)$ of dead-zone decomposition (7) now can be addressed for our stability analysis, as shall be found in the following subsection.

2.4 Stability analysis

Theorem 1 *Under Assumptions 1–2, consider the event-trigger based adaptive control systems (1) with* *the actuator dead-zone input* (3) *consisting of the controller* (27) *and update laws* (26) *and* (28). *The results can be held that all the boundedness of the closed-loop signals is ensured and the asymptotic tracking performance is obtained, i.e.,* $\lim_{t\longrightarrow\infty} z_1(t) = 0$. *Furthermore, the* $\pounds_2$*-norm of the tracking error is guaranteed within an explicit bound.*

Proof With our proposed scheme, an adaptive controller (27) and its event-triggered mechanism (5)–(6) are established for the NCSs with actuator dead-zone. For stability analysis, the final Lyapunov function is chosen as follows:

$$V(t) = \frac{1}{2}z^T z + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta} + \frac{1}{2}\cdot\frac{c}{1+\delta}\tilde{\kappa}^T \Gamma_\kappa^{-1}\tilde{\kappa}, \tag{32}$$

where $z = [z_1, z_2, \ldots, z_n]^T$ is the error variable vector.

Combining with the plant (1) and the definition (13), the derivative of the error variables $z_i (i = 1, 2, \ldots, n)$ can be achieved as

$$\dot{z}_1 = z_2 + \alpha_1 + \varphi_1^T(x_1)\theta \tag{33}$$

$$\dot{z}_i = \dot{z}_{i+1} + \alpha_i + \varphi_i^T(x_1, \ldots, x_i)\theta - \dot{\alpha}_{i-1}, \quad i = 2, 3, \ldots, n-1 \tag{34}$$

$$\dot{z}_n = \varphi_0(x) + \varphi_n^T(x)\theta + D(u) - \dot{\alpha}_{n-1} - y_r^{(n)} \tag{35}$$

With the error variable $z_{n+1} = 0$ and the designed virtual controllers α_i in (15), (19), and (23), the derivative of $V(t)$ is computed as

$$\begin{aligned}
\dot{V}(t) =& z_1(z_2 + \alpha_1 + \varphi_1^T\theta) + \sum_{i=2}^{n} z_i\Big(z_{i+1} + \alpha_i + \varphi_i^T\theta \\
&- \dot{\alpha}_{i-1}\Big) + z_n\left(\varphi_0 + D(u) - y_r^{(n)} - \alpha_n\right) \\
&- \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \frac{c}{1+\delta}\tilde{\kappa}^T\Gamma_\kappa^{-1}\dot{\hat{\kappa}} \\
=& \sum_{i=1}^{n} z_i\alpha_i + \sum_{i=1}^{n} w_i^T\theta z_i + \sum_{i=2}^{n} z_i z_{i-1} \\
&- \sum_{i=2}^{n} z_i \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\tau_n \\
&- \sum_{i=2}^{n} z_i\left(\sum_{j=1}^{i-1}\left[\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(j-1)}}y_r^{(j)}\right]\right) \\
&+ z_n\left(\varphi_0 + D(u) - y_r^{(n)} - \alpha_n\right) \\
&- \tilde{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} - \frac{c}{1+\delta}\tilde{\kappa}^T\Gamma_\kappa^{-1}\dot{\hat{\kappa}} \\
=& -\sum_{i=1}^{n} c_i z_i^2 + z_n\left(D(u) + \rho\right) \\
&+ \tilde{\theta}^T\Gamma^{-1}\left(\Gamma\tau_n - \dot{\hat{\theta}}\right) - \frac{c}{1+\delta}\tilde{\kappa}^T\Gamma_\kappa^{-1}\dot{\hat{\kappa}}
\end{aligned} \tag{36}$$

As mentioned in [28], the inequality is obtained as

$$0 \le |Z| - \frac{Z^2}{\sqrt{Z^2 + \sigma^2}} \le \sigma, \tag{37}$$

where $\sigma \ge 0$. □

Based on Lemma 1 and the Eqs. (7) and (27), the remaining second term in $\dot{V}(t)$ can be rewritten as follows:

$$\begin{aligned}
& z_n\left(D(u) + \rho\right) \\
&= z_n\left(c(u)u + d(u) + \rho\right) \\
&= z_n\rho + z_n d(u) + \frac{c(u)\varpi(t)}{1 + \lambda_1(t)\delta} z_n \\
&\quad - \frac{\lambda_2(t)c(u)m}{1 + \lambda_1(t)\delta} z_n \\
&\le z_n\rho + z_n d(u) + \bar{c}\bar{m}|z_n| + \frac{c\hat{\kappa}^T\nu}{1+\delta} z_n \\
&\le -\frac{c\tilde{\kappa}^T\nu}{1+\delta} z_n + |z_n\rho| - \frac{z_n^2\rho^2}{\sqrt{z_n^2\rho^2 + \epsilon_1(t)^2}} \\
&\quad + \varepsilon\left(|z_n| - \frac{z_n^2}{\sqrt{z_n^2 + \epsilon_2(t)^2}}\right) \\
&\quad + \bar{c}\bar{m}\left(|z_n| - \frac{z_n^2}{\sqrt{z_n^2 + \epsilon_3(t)^2}}\right) \\
&\le -\frac{c\tilde{\kappa}^T\nu}{1+\delta} z_n + \epsilon_1(t) + \varepsilon\epsilon_2(t) + \bar{c}\bar{m}\epsilon_3(t)
\end{aligned} \tag{38}$$

Remark 5 By employing the decomposition model of dead-zone in (26) and the relative threshold event-triggered strategy in (6), a dynamic coefficient $\frac{c(u)}{1+\lambda_1(t)\delta}$ and a disturbance term $\frac{1}{1+\lambda_1(t)\delta}$ are emerged, which obstruct the adaptive control design. To handle such issue, a new adaptive controller is designed with the property $z_n\hat{\kappa}^T\nu \le 0$; furthermore, the dynamic control coefficient and the disturbance term are replaced as its lower bound and its upper bound, respectively, in backstepping procedure.

Thus, $\dot{V}(t)$ is obtained as

$$\dot{V}(t) \leq -\sum_{i=1}^{n} c_i z_i^2 + \epsilon_1(t) + \varepsilon\epsilon_2(t) + \bar{c}\bar{m}\epsilon_3(t) + \tilde{\theta}^T \Gamma^{-1}\left(\Gamma\tau_n - \dot{\hat{\theta}}\right) - \frac{\underline{c}\tilde{\kappa}^T}{1+\delta}\Gamma_k^{-1}\left(\Gamma_k \nu z_n + \dot{\hat{\kappa}}\right) \tag{39}$$

With the update laws (26) and (28), $\dot{V}(t)$ is presented as

$$\dot{V}(t) \leq -\sum_{i=1}^{n} c_i z_i^2 + \epsilon_1(t) + \varepsilon\epsilon_2(t) + \bar{c}\bar{m}\epsilon_3(t) \tag{40}$$

Integrating on both sides of the inequality (40), we have

$$\begin{aligned} V(t) &\leq V(0) + \Delta - \sum_{i=1}^{n} c_i \int_0^T z_i^2 \mathrm{d}t \\ &\leq V(0) + \Delta, \end{aligned} \tag{41}$$

where $\Delta = \int_0^\infty \epsilon_1(t) + \varepsilon\epsilon_2(t) + \bar{c}\bar{m}\epsilon_3(t)\mathrm{d}t$ is regarded as a positive constant. Since the initial value $V(0)$ is bounded, it is concluded that $V(t)$ is bounded too, then $z(t), \tilde{\theta}(t), \tilde{\kappa}(t) \in \pounds_\infty$ are given. Furthermore, based on our assumptions, $y_r, \theta, \kappa \in \pounds_\infty$ are achieved, then $x_1(t), \hat{\theta}(t), \hat{\kappa}(t) \in \pounds_\infty$ are deduced. The continuous function $\varphi_1(x_1)$ and α_i are bounded; thus, $x_2(t) \in \pounds_\infty$ is proved. As previous analysis, $x_3(t), \ldots, x_n(t) \in \pounds_\infty$ can be achieved in sequence. With (27), (29), and (30), the boundedness of the control signal $\varpi(t)$ can be obtained. For now, the conclusion that all closed-loop signals are globally bounded has been proved.

In addition, $z_1(t) \in \pounds_2$ can be derived by (40). By combining with the proved conclusion $\dot{z}_1(t) \in \pounds_\infty$ and employing Barbalat's lemma, the tracking error asymptotically converges to zero can be proved, i.e., $\lim_{t\longrightarrow\infty} z_1(t) = 0$.

The stability analysis is completed already. And the conclusion can be eventually made that the NCSs with actuator dead-zone can maintain steady-state performance and ensure the tracking error asymptotically converges to zero by our proposed scheme.

By integrating the inequality (40) and combining the definition of $V(t)$ in (32), the bound for the $\pounds_2$-norm of the tracking error is obtained as

$$\begin{aligned} \|z_1(t)\|_{2[0,T]} &= \sqrt{\int_0^T z_1^2(t)\mathrm{d}t} \\ &\leq \frac{1}{\sqrt{2c_1}}\Big(\tilde{\theta}^T(0)\Gamma^{-1}\tilde{\theta}(0) \\ &\quad + \frac{\underline{c}}{1+\delta}\tilde{\kappa}^T(0)\Gamma_\kappa^{-1}\tilde{\kappa}(0) + 2\Delta\Big)^{\frac{1}{2}} \end{aligned} \tag{42}$$

Now, the proof of the Theorem 1 is completed.

Remark 6 The $\pounds_2$-norm of the tracking error is guaranteed within an explicit bound, as shown in (42), such that the transient performance is able to improve by tuning the control coefficients c_1, Γ, Γ_κ.

3 Simulation results

3.1 A numerical example

In this subsection, a numerical example is presented to verify the previous methodology for the NCSs with dead-zone nonlinearity.

Consider the uncertain nonlinear system defined globally as

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^2\theta \\ \dot{x}_2 &= x_1x_2 + \frac{1-e^{-x_2}}{1+e^{-x_2}}\theta + D(u), \end{aligned} \tag{43}$$

where $\theta \in \Re$, $D(u)$ represents the dead-zone input.

For simulation, the dead-zone parameters in (3) are chosen as follows:

$$\rho_r(t_r) = 1.1, \rho_l(t_l) = 0.9, b_r = 0.6, b_l = -0.8 \tag{44}$$

With our proposed scheme, the controller and its event-triggered mechanism are co-designed as (27) and (6), respectively. The parameters of triggering event are given as $\delta = 0.8$, $m = 0.2$. Besides, the other parameters of the controller are set as $x_1(0) = 0.2$, $x_2(0) = 0$, $\theta = 1$, $c_1 = c_2 = 5$, $\Gamma = 3$, $\Gamma_\kappa = 3.2$, and the positive integrable functions are defined as $\epsilon_i(t) = 0.1e^{-0.1t}$ for $i = 1, 2, 3$. The reference signal is $y_r = \sin(0.1t)$.

The control objective is to guarantee the tracking performance of the hybrid system with dead-zone input; furthermore, the tracking error can be ensured asymptotic approaching to zero, i.e., $\lim_{t\rightarrow\infty}(y - y_r) = 0$.

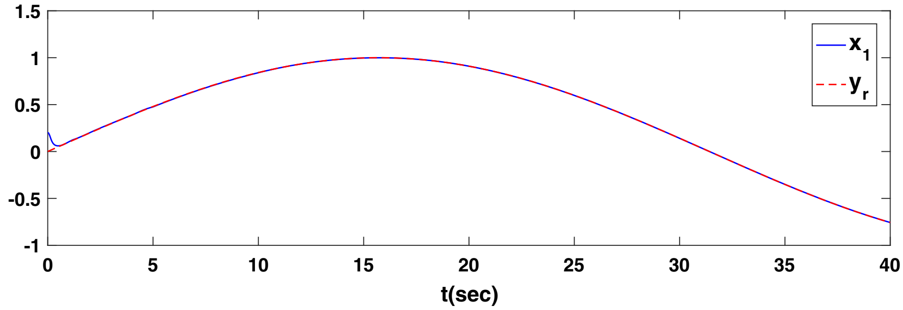


Fig. 3 Tracking performance

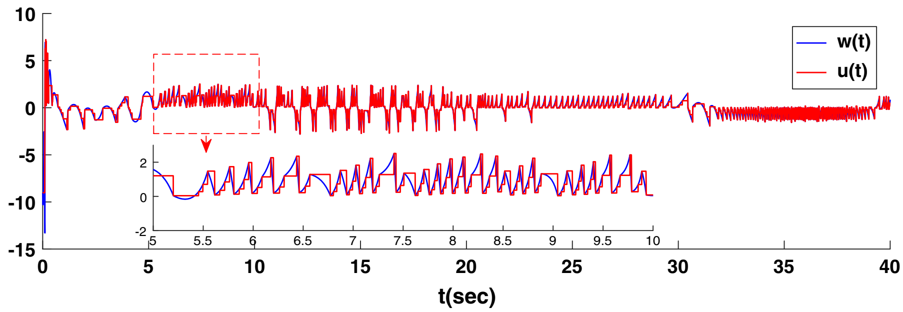


Fig. 4 Designed controller $w(t)$ and control signal $u(t)$

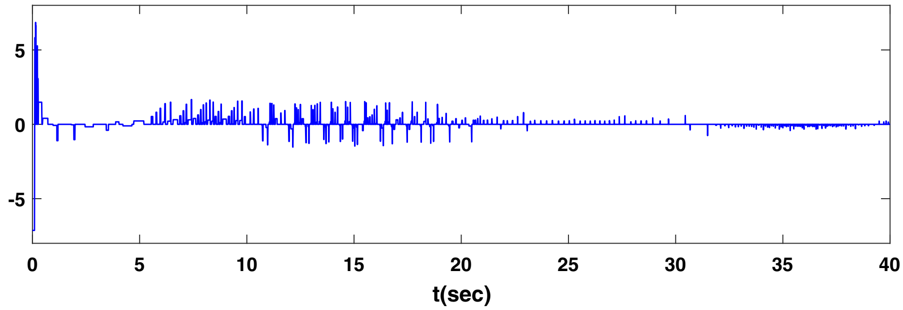


Fig. 5 Control signal $u(t)$ suffers from dead-zone

With the above parameters, an event-triggered robust adaptive controller is constructed by utilizing the proposed scheme, and the simulation results are shown in Figs. 3, 4, 5, and 6. Figure 3 shows the tracking performance between the system output x_1 and the reference signal y_r. In Fig. 4, the controller $w(t)$ and the control signal $u(t)$ are obtained, while the dead-zone phenomenon of the input signal $u(t)$ is presented in Fig. 5. In addition, as shown in Fig. 6, the tracking error $e(t)$ is ensured asymptotic approaching to zero, i.e., $\lim_{t\longrightarrow\infty} e(t) = 0$.

Remark 7 1. Dead-zone phenomenon can be observed from Fig. 5, i.e., the control signal $u(t)$ suf-

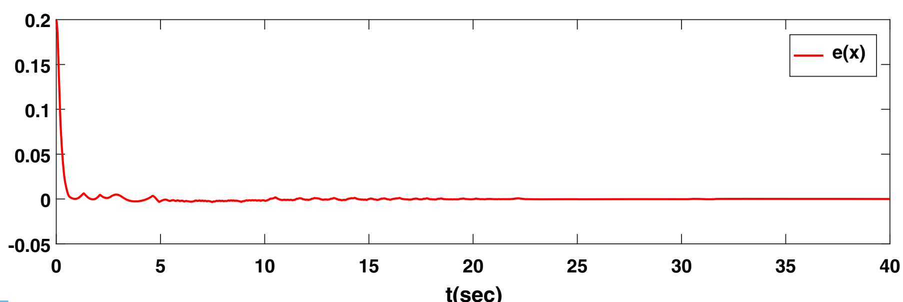


Fig. 6 The tracking error $e(t)$

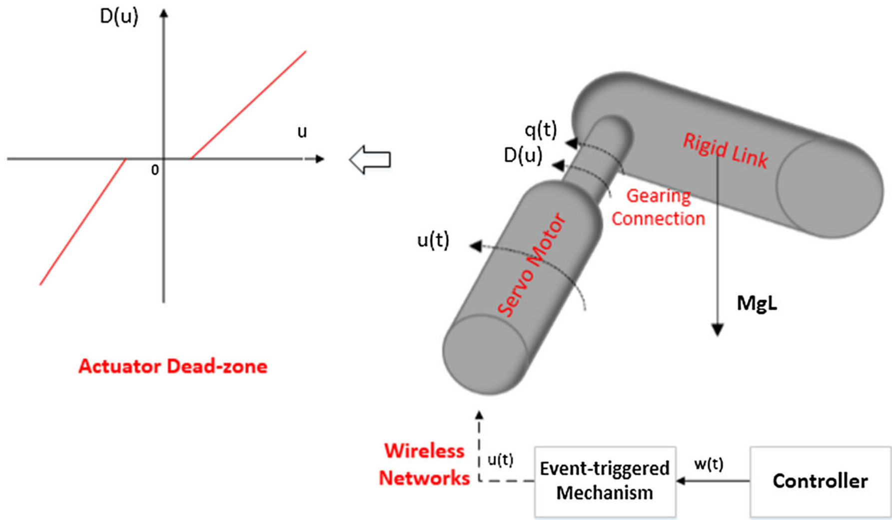


Fig. 7 Robotic manipulator system

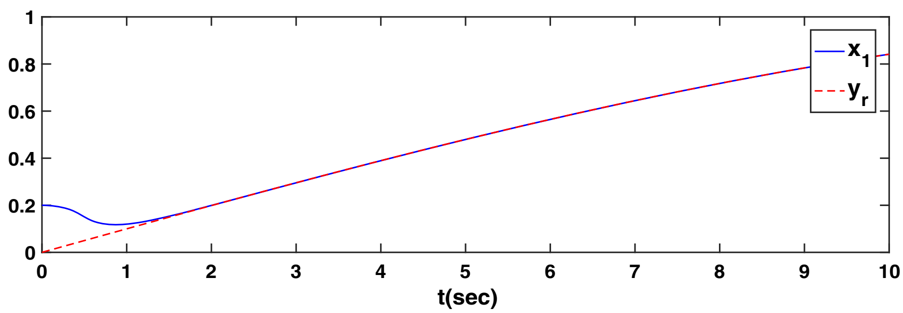


Fig. 8 Tracking performance

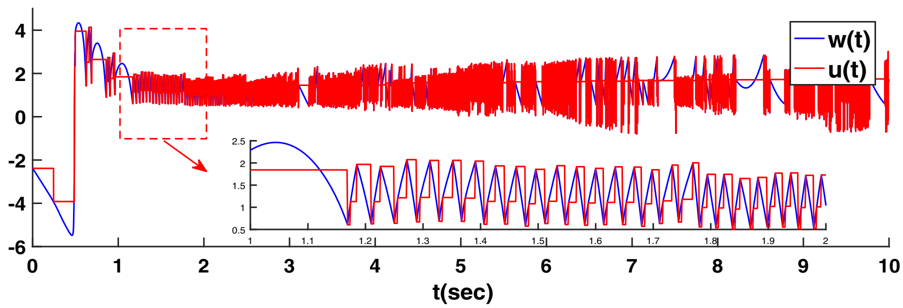


Fig. 9 Designed controller $w(t)$ and control signal $u(t)$

fers from dead-zone nonlinearity in actuator. By applying our proposed scheme to the considered system, the dead-zone effect can be compensated and a perfect performance of the tracking error can be obtained.

2. As previous descriptions, the transmissions between control signal and dynamic systems are usually shared by limited bandwidth. To reduce the communication rate, an event-triggered strategy is employed in our control design. The effectiveness of such scheme is presented by comparing the two signals in Fig. 4.
3. In contrast to the scheme in [31], the proposed methodology in this paper also ensures that the tracking error asymptotically converges to zero. The simulation presented in Fig. 6 verifies the established theoretical results in Theorem 1.

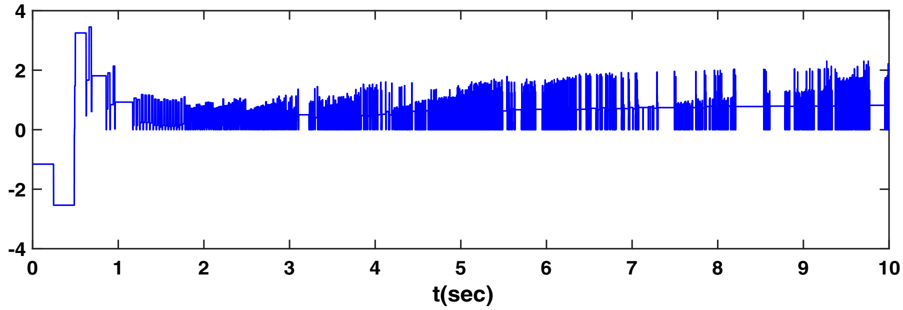


Fig. 10 Control signal $u(t)$ suffers from dead-zone

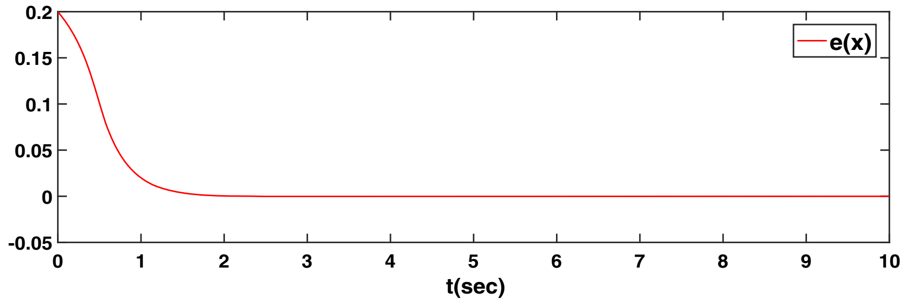


Fig. 11 The tracking error $e(t)$

3.2 A practical example

A network control-based robotic manipulator system with dead-zone nonlinearity is considered in this subsection for simulation. For the visualization, the practical model is shown in Fig. 7. The dynamic of the robotic model is given as below.

$$J\ddot{q}(t) + B\dot{q}(t) + MgL \times \sin(q(t)) = D(u), \qquad (45)$$

where $x_1 = q$ and $x_2 = \dot{q}$ denote the angle and angular velocity of the rigid link, respectively. J is the rotation inertia of the servomotor, and B represents the damping coefficient. The link's total mass is defined as M, and the length from the joint axis to the mass center is denoted as L. g is the value of gravitational acceleration.

For simulation, the parameters are defined as $J = 1$, $MgL = 10$, $B = 2$, the initial values and the design parameters are chosen as $x_1(0) = 0.8$, $x_2(0) = 0$, $c_1 = c_2 = 0.6$, $\gamma = 2$, $\Gamma = 2$, $\delta_1 = \delta_2 = 0.38$, and the parameters of triggering event are set as $\delta = 0.8$, $m = 0.1$. $D(u)$ is regarded as dead-zone input of the system, whose parameters are given as the same as (44).

The control objective of this robotic manipulator is to ensure the output accurately tracks the desired trajectory $y_r = \sin(0.1t)$. By applying our proposed control approach to the system, the results are shown in Figs. 8, 9, 10, and 11. It can be seen from Fig. 8 that the angle x_1 tracks the reference signal y_r perfectly. In Fig. 9, the designed controller $w(t)$ and the control signal $u(t)$ are presented, which implies that our proposed scheme is feasible to reduce communication rate during signal transmission. The dead-zone nonlinearity is occurred in gearing connection, which is shown in Fig. 10. Finally, it is exhibited in Fig. 11 that the tracking error $e(t)$ asymptotically converges to zero, which verifies our assumption and illustrates the superiority of our scheme.

From the above observations, the proposed adaptive controller successfully suppresses the nonsmooth actuator effects and event-triggered effects and guarantees an asymptotic tracking performance.

4 Conclusion

The problem of event-triggered robust adaptive tracking control for the NCSs with dead-zone input is considered. By constructing a new decomposition model of dead-zone nonlinearity and utilizing the relative threshold event-triggered strategy, a new controller is established in this paper to solve the problem. It is proved that all the closed-loop signals are globally bounded and the excellent asymptotic tracking performance is ensured by our proposed approach. Besides, an explicit bound for the $\mathcal{L}_2$-norm of the tracking error is obtained to guarantee the transient performance with our scheme.

However, the event-triggered tracking control problem for nonparametric strict-feedback system still remains as a question for future research, which is possible to combine with the results in [3,4,19,26–28] for solving. Besides, the transmission problem mentioned in [14] needs to be investigated further, which considers another wireless network connection between plant and controller. These will be significant topics for future research.

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